ST. ALOYSIUS COLLEGE (AUTO), JABALPUR
DEPARTMENT OF MATHEMATICS 2021-22
M. Sc. (MATHEMATICS)
SECOND SEMESTER
CORE PAPER I: ADVANCED ABSTRACT ALGEBRA-II

CREDIT: 5
MAX MARKS: 40
MIN MARKS: 14

COURSE OBJECTIVE:

Give the students experience, knowledge, and confidence to move forward in the study of mathematics.

COURSE LEARNING OUTCOMES:

Upon successful completion of this course, the student will be able to:

CLO1 Impart knowledge of some fundamental results and techniques from the Galois theory.

CLO2 Analyze and demonstrate examples of modules and rings,

CLO3 Discuss general properties of Modules, Sub modules and Quotient module.

CLO4 Confer knowledge of Noetherian, Artinian modules and rings.

CLO5 Appreciate the concepts of finitely generated module over principal ideal domain and its application to finitely generated Abelian groups.

Unit-I: The element of Galois theory: Automorphism of a Field, Group of Automorphisms of a Field, Fixed field and its theorems, Normal Extension and its theorems, Fundamental theorem of Galois theory.

Unit-II: Introduction of Modules, Examples, General properties of Modules, Submodules and Direct Sum of Submodules, R-homomorphisms and Quotient Modules.

Unit-III: Finitely generated modules, Cyclic modules, Simple modules, Semi-simple modules, Schur's lemma, Free Modules, Rank of a Module.

Unit-IV: Noetherian and Artinian Modules: Examples and theorems, Ascending and Descending Chain Condition (acc and dcc), Noetherian and Artinian Rings, Examples, Hilbert Basis theorem.

Unit-V: Finitely Generated Modules over a Principal Ideal Domain, Fundamental Structure theorem of Finitely Generated Modules over a Principal Ideal Domain, Applications to Finitely Generated Abelian Groups.

Text Books :

1. I.N. Herstein, Topics in Algebra, Wiley Eastern Ltd., New Delhi, 1975 (For Units II, III, IV,V).
2. A.R.Vasishtha, Modern Algebra, Krishna Prakashan Mandir, Meerut (U.P.) (For Unit I).
3. P.B. Bhattacharya, S.K. Jain and S.R. Nagpaul, Basic Abstract Algebra (2nd Ed.), Cambridge University Press, Indian Edition, 1997 (For Units II, III, IV,V).

Reference Books:

1. Vivek Sahai and Vikas Bist, Algebra, Narosa Publishing House, 1999.
2. I.S. Luther and I.B.S. Passi, Algebra, Vol. I, Groups, Narosa Publishing House, 1996.
3. Surjeet Singh and Quazi Zameeruddin, Modern Algebra, Vikas Publishing House Pvt. Ltd., 1990.
4. N. Jacobson, Basic Algebra, Vol. I & II, Hindustan Publishing Company, 1980.

5. S. Lang, Algebra, 3rd Edition, Addition-Wesley, 1993.
6. Ramji Lal, Algebra, Vol. I & II, Shail Publication, 2002.

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SECOND SEMESTER
CORE PAPER II : REAL ANALYSIS – II

CREDIT: 5
MAX MARKS: 40
MIN MARKS: 14

COURSE OBJECTIVE:

To make the students see and understand the connection and transition between previously studied length of sets concepts and more advanced length concept viz. Measure.

COURSE LEARNING OUTCOMES:

Upon successful completion of this course, the student will be able to:

CLO1 The basic tools, concepts and results of measure theory and appreciate its importance in the wider context of mathematics.

CLO2 The knowledge of null sets, outer measure, measurable sets and Lebesgue measure, sigma-fields.

CLO3 Introduce many mathematical concepts studied in mathematics such as the Lebesgue integration, Fatou's Lemma, Monotone and Dominated Convergence Theorems.

CLO4 Understand the concept of Convex function and its geometrical representation

CLO5 Understand the concept of Dual space and convergence in measure

Unit-I: Lebesgue outer measure. Measurable sets. Regularity. Measurable functions. Borel and Lebesgue measurability. Non-measurable sets.

Unit-II: Integration of Non-negative functions. The General integral. Integration of Series, Reimann and Lebesgue Integrals.

Unit-III: The Four derivatives. Functions of Bounded variation. Lebesgue Differentiation Theorem, Differentiation and Integration.

Unit-IV: The LP-spaces, Convex functions, Jensen's inequality. Holder and Minkowski inequalities. Completeness of LP.

Unit-V: Dual of space when $1 \leq P < \infty$, Convergence in Measure, Uniform Convergence and almost Uniform Convergence.

Text Book: G. de Barra. Measure Theory and Integration, Wiley Eastern (Indian Ed.).

Reference Books:

1. Walter Rudin, Principles of Mathematical Analysis, McGraw-Hill, International student edition.
2. H.L. Royden, Real Analysis, Macmillan, Indian Edition.

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SECOND SEMESTER
CORE PAPER III: ORDINARY AND PARTIAL DIFFERENTIAL EQUATION

CREDIT: 5
MAX MARKS: 40
MIN MARKS: 14

COURSE OBJECTIVE:

To make the students see and understand the ordinary and partial differential equation and also different methods to solve differential equation at ordinary and singular points.

COURSE LEARNING OUTCOMES:

Upon successful completion of this course, the student will be able to:

CLO1. Classify ordinary and partial differential equation.

CLO2. Use power series method to solve differential equations about ordinary points.

CLO3. Use the Method of Frobenius to solve differential equations about regular singular points.

CLO4. Evaluate Laplace transform of a function its inverse. Find the Laplace transform of derivatives, integrals and periodic functions.

CLO5. Apply Laplace transforms to solve initial-value problems for linear differential equations with constant coefficients.

UNIT I Exact differential equations and adjoints , The adjoint operator, Lagrange's identity , Strum-Liouville differential equation , Eigen values , The normal form, change of independent's variable, Lagrange's method of variation of parameters.

UNIT II Partial differential equation, Construction of partial differential equation of first order, Lagrange's linear equation, Charpit's general method of solutions, Green's functions, Domain and range of the operators, One dimensional Green's functions, Construction of Green's functions.

UNIT III Power series solution and special functions, A review of power series, Series solution of first order linear equations, Second order linear equations, Ordinary points, Regular singular points, Gauss's hypergeometric series.

UNIT-IV Laplace Transforms, Integral transforms, A few remarks on the theory, Conditions for the existence of Laplace transforms, Applications to differential equations.

UNIT-V Derivatives and intergrals of Laplace transforms, Convolutions and Abel's Mechanical problem. More about convolutions, The unit step and impulse functions.

Text Books:

1. G.F.Sinmons, Differential Equation with applications and Historical Notes, McGraw Hill international Editions, 1991(for Units IV&V)
2. B.P Parashar; Differential and Integral Equations, CBS publishers and Distributors Ltd. 1992(for Units I,II&III).

Reference Books:

1. H.T.H. Piaggio, An Elementary Treatise on differential Equations and Their Applications, Indian Reprint,1966.
2. E.A. Coddington, An Introduction, The Solution of Ordinary Differential Equations, Indian reprint.
3. B.L.Ince and I.N.Sneddon, The Solution of Ordinary Differential Equations,Longman.1987.
4. Ian Sneddon, Elements of Partial Differential Equations, McGraw Hill International Editions, 1957

Pratibha Hadang. Nandini Singh

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SECOND SEMESTER
CORE PAPER IV : COMPLEX ANALYSIS II

CREDIT: 5
MAX MARKS: 40
MIN MARKS: 14

COURSE OBJECTIVE:

Learn techniques of analysis that make practical problems easy (e.g. graphical rotation and scaling as an example of complex multiplication) , Appreciate how mathematics is used in design(e.g. conformal mapping).

COURSE LEARNING OUTCOMES:

Upon successful completion of this course, the student will be able to:

CLO 1 Perform basic algebraic manipulation with complex numbers
CLO 2 Understand the geometric interpretation of complex numbers
CLO 3 Know methods of finding the nth roots of complex numbers and the solutions of simple polynomial equations.
CLO 4 Use analytical functions and conformal mappings;
CLO 5 Compute definite integrals using residue calculus;

UNIT 1. Weierstrass factorization theorem, Gamma function, Euler's Gamma function, Properties of Gamma function, Gauss' Formula, Functional Equation, The Riemann Zeta function, Extention of Zeta Function, Riemann's functional Equation.

UNIT 2. Mittage-Leffler's theorem, Analytic Continuation, Uniqueness of Analytic Continuation, Power series Method of Analytic Continuation, Schwarz's Reflection Principal for Symmetric Region.

UNIT 3. Harmonic functions, Basic Properties of Harmonic Functions, Harmonic Conjugates, Mean-Valued Theorem for Harmonic Functions, Harmonic Functions on a Disc, Poisson Kernel, Proposition, Harnack's inequality.

UNIT 4. Calculus of Residues, Evaluation of Certain integrals, Integral of type Case of poles on the Real Axis(Indenting Method), Integrals of Many Valued Function such as Z^a , log Z, A Quadrant or a sector of a circle as the contour.

UNIT 5. Conformal mappings,Sufficient condition for w=f(z) to represent a Conformal Mapping,Necessary condition for w=f(z) to represent a Conformal Mapping, Certain type of transformations, Translation $w=z+\propto$, rotation only, Magnification only,Magnification and rotation both,Translation, Magnification and Rotation, Rotation and Inversion.

Text Book :

J.B. Convey, Function of one complex variable, Springer-Verlag, 1980.

Reference Books:

1. S. Ponnuswamy, Foundation of complex analysis, Narosa Publishing House,1997.

2. L.V. Ahlfors, complex analysis,McGraw Hill,1979.

3. E. C. Tichmarsh, The theory of functions, Oxford University, Press, London.

4. Dr. H.K.Pathak, Complex Analysis

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SECOND SEMESTER
CORE ELECTIVE PAPER V(A) : NUMBER THEORY

CREDIT: 5
MAX MARKS: 40
MIN MARKS: 14

COURSE OBJECTIVE:

To expose students Number Theory as a beautiful subject of Mathematics because of its treasure of fascinating problems and academic appeal. To understand the idea behind the famous quotation of Gauss on number theory. To allow students to experience mathematics as imaginative, empirical science.

COURSE LEARNING OUTCOMES:

Upon successful completion of this course, the student will be able to:

CLO1. Understand effective expression of concepts and results in number theory.

CLO2. Construct mathematical proofs of statements and explore counter examples to false statements.

CLO3. Understand the logic and techniques behind the major proofs in number theory.

CLO4. Gain perception of numerous applications in mathematics as well as in practical applications like cryptography of results from number theory.

CLO5. Attempt many unsolved problems in number theory.

Unit-I: Divisibility theory in integers: Division algorithm, Greatest common divisor, Euclidean algorithm.
Primes and their distribution: The fundamental theorem of arithmetic, the sieve of Eratosthenes, the Goldbach conjecture.

Unit-II: The theory of congruence: Basic properties of congruences, binary and decimal representations of integers.

Fermat's Little theorem and pseudoprimes, Wilson's theorem, the Fermat-Kraitchik factorisation method, Chinese Remainder Theorem.

Unit-III: Number-Theoretic Functions: $\varphi(n)$, $d(n)$, $\sigma(n)$, $\mu(n)$ and their properties, Mobius inversion formula, the greatest integer function, Euler' theorem.

Unit-IV: Primitive roots: The order of an integer modulo n, Lagrange's theorem, composite numbers having primitive roots, the theory of indices.

Euler's criterion, Legendre's symbol and its properties, Quadratic reciprocity Law.

Unit-V: Introduction to Cryptography: From Caesar cipher to public key cryptography, the Knapsack cryptosystem, an application of primitive roots to cryptography.

Numbers of special form: Mersenne primes and perfect numbers.

Certain Diophantine equations: $ax + by = c$, $x^2 + y^2 = z^2$, Fermat's Last Theorem (without proof)

Text book –

1. Burton, D.M., *Elementary Number Theory, 7th Edition*, MacGraw-Hill Education, 2010 .

Reference Books:

1. Niven I., Zuckerman, H.S. and Montogomery, H.L., *Introduction to Theory of Numbers, 5th Edition*, John Wiley & Sons, 1991.

2. Hardy, G.H., and Wright, E.M., *Introduction to Theory of Numbers*, *4th Edition*, Oxford University Press, 1991.

3. Silverman, J.H., *A Friendly Introduction to Number Theory, 3rd Edition*, Pearson, 2009.

4. Robbins, N., *Beginning Number Theory, 2nd Edition*, Jones and Bartlett Publishers, 2006.

5. Chowdhury, K.C., *A First Course in Theory of Numbers*, Asian Books Pvt. Ltd., 2007.

6. Stewart, B. M., Theory of Numbers, The Mac-millan copany, 1964.

7. Koshy T. Elementary Number Theory, Academic Press , 2007.

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SECOND SEMESTER
CORE ELECTIVE PAPER V(B): TOPOLOGY-II

CREDIT: 5
MAX MARKS: 40
MIN MARKS: 14

Course Objective:

To train the students in the area of topology. To give adequate knowledge of the subject that can be used by the students for further applications in their respective sphere of interest.

COURSE LEARNING OUTCOMES:

Upon successful completion of this course, the student will be able to:

CLO 1 Acquisition of knowledge in various topics of topology.

CLO 2 Appreciation of beauty of profound mathematical results such as Heine-Borel theorem and Urysohn's Lemma.

CLO 3 Understanding the dynamics of methods of mathematical proof.

CLO 4 Understanding the Nets, Filters, Convergence, Ultra Filters and Compactness.

CLO 5 Understanding the fundamental group, covering spaces and Homotopy of Paths.

Unit –I: Continuous functions and compact sets, Basic properties of Compactness, Compactness and Finite Intersection Property, Product of spaces, Projection maps, Tychonoff's theorem, Local compactness, Heine Boral theorem.

Unit – II: Separation: T_1 –speces, Hausdorff spaces, Regular, Completely regular spaces and Normal spaces, their characterization and basic properties (As given in the Chapter-V of G.F. Simmons Book)

Unit – III: Urysohn's Lemma, Tietze extension theorem, one-point compactification, the stone-cechcompactification, Urysohn Metrization theorem (As given in the Chapter-V of G.F. Simmons Book)

Unit – IV: Net's & Filters, Topology and convergence of Nets, Housdorffness& Nets, Compactness & Nets, Filters and Convergence, Ultra Filters and Compactness.

Unit – V: The fundamental group and covering spaces, Homotopy of Paths, The Fundamental Group, Homomorphism, Covering spaces, The Fundamental Group of the circle.

Text Book:

1. James R. Munkres, Topology, A First Course, Prentice Hall of India Pvt. Ltd. New Delhi.
2. G. F. Simmons, Topology and Modern Analysis , McGraw Hill International Edition, 1963.

Reference Books:

1. K. D. Joshi, Introduction to General Topology, Wiley Eastern Ltd, New Delhi.
2. J. L. Kelly, General Toplogy, Van Nostrand, New York, 1995.
3. K. Chandra Shekhara Rao, Topology, Narosa Publications.
4. J. P. Chauhan and J. N. Sharma, Krishana Publications.